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A SHORT NOTE ON MATRICES IN INTERVAL VALUED INTUITIONISTIC FUZZY SOFT SET THEORY AND THEIR APPLICATION IN PREDICTING ELECTION RESULTS

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Abstract: In this paper we give a brief discussion on different types of interval valued intuitionistic fuzzy soft matrices and apply some new matrix operations on them. Moreover a new methodology has been developed to solve interval valued intuitionistic fuzzy soft set based real life decision making problems which may contain more than one decision maker and put an effort to apply it to a more relevant way in predicting election results by using the concept of choice matrix.
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1. Introduction: To tackle those problems which we have experienced in our day to day life becomes complex more and more. And most of them deals with uncertainties i.e. in Mathematics the term ‘vague’ is usually used for such cases. So we need some effective measures or tools which are capable to solve such problems. Some tools such as fuzzy set[13], L-fuzzy set[5], intuitionistic fuzzy set[1], interval valued fuzzy set[12], interval valued intuitionistic fuzzy set[2], rough set[11] etc are introduced earlier.

But due to the more complexity of the modern days problems, sometimes it is difficult to determine the membership or non membership value for each and every case. For this purpose, later on, another mathematical tool known as soft set is introduced by Molodtsov[10] in 1999. Soft set theory basically used for parametrization in a data.

Many researchers and Mathematicians used soft set theory in multiple directions. Which leads to the notion of fuzzy soft set[8], intuitionistic fuzzy soft set[9], interval valued fuzzy soft set[4], interval valued intuitionistic fuzzy soft set[7] etc.

Moreover to reduce parameter set of a soft set rough set is used. To store and manipulate data in a computer [3] have introduced the definition of soft matrices which are representations of soft sets.It has several advantages.

In this article we have proposed the notion of interval valued intuitionistic fuzzy soft matrix. Then we have defined its types with suitable examples. Here we have also proposed the concept of choice matrix associated with an interval valued intuitionistic fuzzy soft set. Furthermore we have introduced some new operations on interval valued intuitionistic fuzzy soft matrices and choice matrices. Then based on some of these new matrix operations a new efficient solution technique has been developed to solve interval valued intuitionistic fuzzy soft set based real life decision making problem which may contain more than one decision maker. The novelty of the new approach is that it may solve any interval valued intuitionistic fuzzy soft set based decision making problem involving large number of decision makers very easily and the computational procedure is also very simple. Finally to realize this newly proposed methodology we apply it to predict election results.

2. Preliminaries: Some important definitions with examples are discussed in the following.

Definition 2.1:[10] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$.

Example 2.1: Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7\}$ is a universe consisting of seven houses and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ is a set of parameters considered by the decision makers where e_1, e_2, e_3 and e_4 represent the parameters 'beautiful', 'modern', 'cheap', 'in green surroundings' respectively. Now, we consider a soft set (F, A) which describes the 'attractiveness of the house' that Mr. X is going to buy. In this case, to define the soft set (F, A) means to point out beautiful houses, modern houses and so on. Consider the mapping F given by 'houses (.)' where (.) is to be filled by one of the parameters $e_i \in A$. For instance, $F(e_1)$ means 'houses (beautiful)' and the functional value is the set consisting of all the beautiful houses in U.

Let $F(e_1) = \{h_5, h_7\}$

$F(e_2) = \{h_1, h_4, h_6, h_7\}$

$F(e_3) = \{h_1, h_3\}$

$F(e_4) = \{h_2, h_4, h_5\}$.

Tabular representation of the soft set (F, A) is given by

	h_1	h_2	h_3	h_4	h_5	h_6	h_7
e_1	0	0	0	0	1	0	1
e_2	1	0	0	1	0	1	1
e_3	1	0	1	0	0	0	0
e_4	0	1	0	1	1	0	0

Definition 2.2: [8] Let U be an initial universe and E be a set of parameters. Let I^U be the set of all fuzzy subsets of U and $A \subseteq E$. Then the pair (F, A) is called a *fuzzy soft set* over U, where F is a mapping given by $F : A \to I^U$.

Example 2.2: Suppose that U is the set of houses under consideration, E is the set of parameters where each parameter is a fuzzy word or a sentence involving fuzzy words, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ where e_1, e_2, e_3, e_4, e_5 e_6 e_7 represent the parameters 'expensive', 'beautiful', 'wooden', 'cheap', 'in green surroundings', 'modern', 'in good repair' respectively. In this case, to define a fuzzy soft set means to point out expensive houses, beautiful houses and so on.

Suppose there are six houses in the universe U given by $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $A = \{e_1, e_2, e_3, e_4, e_5\}$ is a set of parameters considered by the decision makers. The fuzzy soft set (F, A) describes "the attractiveness of the houses" which Mr. X is going to buy.

Let $F(e_1) = \{(h_1, 0.5), (h_2, 1.0), (h_3, 0.4), (h_4, 1.0), (h_5, 0.3), (h_6, 0.0)\}$

$F(e_2) = \{(h_1, 1.0), (h_2, 0.4), (h_3, 1.0), (h_4, 0.4), (h_5, 0.6), (h_6, 0.8)\}$

$F(e_3) = \{(h_1, 0.2), (h_2, 0.3), (h_3, 1.0), (h_4, 1.0), (h_5, 1.0), (h_6, 0.0)\}$

$F(e_4) = \{(h_1, 1.0), (h_2, 0.0), (h_3, 1.0), (h_4, 0.2), (h_5, 1.0), (h_6, 0.2)\}$

$$F(e_5)=\{(h_1,1.0),(h_2,0.1),(h_3,0.5),(h_4,0.3),(h_5,0.2),(h_6,0.3)\}$$

Here the mapping F given by 'houses (.)' where (.) is to be filled by one of the parameters $e_i \in A$. Therefore $F(e_1)$ means "houses (expensive)" whose functional value is the fuzzy set $\{(h_1,0.5),(h_2,1.0),(h_3,0.4),(h_4,1.0),(h_5,0.3),(h_6,0.0)\}$

Thus we can view the fuzzy soft set (F,A) as a collection of fuzzy approximations (which are fuzzy sets) as below:

$$(F,A)=\{\text{exp}\,ensive\;houses=\{(h_1,0.5),(h_2,1.0),(h_3,0.4),(h_4,1.0),(h_5,0.3),(h_6,0.0)\}$$
$$beautiful\;houses=\{(h_1,1.0),(h_2,0.4),(h_3,1.0),(h_4,0.4),(h_5,0.6),(h_6,0.8)\}$$
$$wooden\;houses=\{(h_1,0.2),(h_2,0.3),(h_3,1.0),(h_4,1.0),(h_5,1.0),(h_6,0.0)\}$$
$$cheap\;houses=\{(h_1,1.0),(h_2,0.0),(h_3,1.0),(h_4,0.2),(h_5,1.0),(h_6,0.2)\}$$
$$in\;green\;surroundings=\{(h_1,1.0),(h_2,0.1),(h_3,0.5),(h_4,0.3),(h_5,0.2),(h_6,0.3)\}\}$$

Definition 2.3 [**9**] Let U be an initial universe and E be a set of parameters. Let IF^U be the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then the pair (F,A) is called an *intuitionistic fuzzy soft set* over U, where F is a mapping given by $F: A \to IF^U$.

Example 2.3: Consider an intuitionistic fuzzy soft set (F,A) where U is a set of five houses under the consideration of a decision maker to purchase, which is denoted by $U=\{h_1,h_2,h_3,h_4,h_5\}$ and A is a parameter set where $A=\{e_1,e_2,e_3,e_4,e_5\}$ ={expensive, beautiful, wooden, in good repair, in the green surroundings}. The intuitionistic fuzzy soft set (F,A) describes the "attractiveness of the houses" to the decision maker.

$$F(e_1)=\{(h_1,0.6,0.1),(h_2,0.8,0.05),(h_3,0.6,0.2),(h_4,0.65,0.15),(h_5,0.56,0.2)\}$$
$$F(e_2)=\{(h_1,0.7,0.15),(h_2,0.6,0.15),(h_3,0.5,0.2),(h_4,0.7,0.15),(h_5,0.6,0.25)\}$$
$$F(e_3)=\{(h_1,0.75,0.1),(h_2,0.5,0.2),(h_3,0.6,0.1),(h_4,0.68,0.1),(h_5,0.7,0.1)\}$$
$$F(e_4)=\{(h_1,0.8,0.01),(h_2,0.65,0.2),(h_3,0.66,0.2),(h_4,0.69,0.1)(h_5,0.72,0.1)\}$$
$$F(e_5)=\{(h_1,0.77,0.05),(h_2,0.6,0.2),(h_3,0.6,0.2),(h_4,0.63,0.15),(h_5,0.7,0.1)\}$$

The intuitionistic fuzzy soft set (F,A) is a parameterized family $\{F(e_i, i=1,2,3,4,5)\}$ of interval valued intuitionistic fuzzy sets on U and

$$(F,A)=\{\text{exp}\,ensive\;houses=\{(h_1,0.6,0.1),(h_2,0.8,0.05),(h_3,0.6,0.2),(h_4,0.65,0.15),(h_5,0.56,0.2)\},$$
$$beautiful\;houses=\{(h_1,0.7,0.15),(h_2,0.6,0.15),(h_3,0.5,0.2),(h_4,0.7,0.15),(h_5,0.6,0.25)\}$$
$$wooden\;houses=\{(h_1,0.75,0.1),(h_2,0.5,0.2),(h_3,0.6,0.1),(h_4,0.68,0.1),(h_5,0.7,0.1)\}$$
$$\text{in good repair houses}=\{(h_1,0.8,0.01),(h_2,0.65,0.2),(h_3,0.66,0.2),(h_4,0.69,0.1)(h_5,0.72,0.1)\}$$
$$\text{in the green surroundings}=\{(h_1,0.77,0.05),(h_2,0.6,0.2),(h_3,0.6,0.2),(h_4,0.63,0.15),(h_5,0.7,0.1)\}$$

Definition 2.4: [12] Let U be an initial universe and E be a set of parameters, a pair(F,E) is called an *interval valued- fuzzy soft set* over *F*(*U*), where F is a mapping given by *F*:*E* → *F*(*U*), where *F*(*U*) is the set of all interval-valued fuzzy sets of U.

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of *U*, thus, its universe is the set of all interval-valued fuzzy sets of *U*, i.e. *F*(*U*). An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to *F(U)*, $e \in E$, $F(U)$ is referred as the interval fuzzy value set of parameters e, it is actually an interval-valued fuzzy set of U where $x \in U$ and $e \in E$, it can be written as:

$F(e)=\{(x,\mu_{F(e)}(x)):x\in U\}$ where $F(U)$ is the interval-valued fuzzy membership degree that object x holds on parameter.

Example 2.4: we consider $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$ is the set of houses under consideration and A is the set of parameters and $A=\{\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4\}$ = {beautiful, wooden, cheap, in the green surroundings}. The tabular representation of an interval-valued fuzzy soft set (*F*,*A*) is shown below in Table .

In Table , we can see that the precise evaluation for each object on each parameter is unknown while the lower and upper limits of such an evaluation is given. For example, we cannot present the precise degree of how beautiful house h_1 is, however, house h_1 is at least beautiful on the degree of 0.7 and it is at most beautiful on the degree of 0.9.

An interval-valued fuzzy soft set (*F*,*A*) :

U	ε_1	ε_2	ε_3	ε_4
h_1	[0.7, 0.9]	[0.6, 0.7]	[0.3, 0.5]	[0.5, 0.8]
h_2	[0.6, 0.8]	[0.8, 1.0]	[0.8, 0.9]	[0.9, 1.0]
h_3	[0.5, 0.6]	[0.2, 0.4]	[0.5, 0.7]	[0.7, 0.9]
h_4	[0.6, 0.8]	[0.0, 0.1]	[0.7, 1.0]	[0.6, 0.8]
h_5	[0.8, 0.9]	[0.1, 0.3]	[0.9, 1.0]	[0.2, 0.5]
h_6	[0.8, 1.0]	[0.7, 0.8]	[0.2, 0.5]	[0.7, 1.0]

Definition 2.5: [7] Let U be an initial universe and E be a set of parameters. Let $IVIFS^U$ be the set of all interval valued intuitionistic fuzzy sets on U and $A \subseteq E$. Then the pair (F,A) is called an *interval valued intuitionistic fuzzy soft set (IVIFSS for short)* over U, where F is a mapping given by $F : A \to IVIFS^U$.

Example 2.5: Let us consider the above example with (F,A) as an interval valued intuitionistic fuzzy soft set. Let

$$F(e_1)=\{(h_1,[0.6,0.8],[0.1,0.2]),(h_2,[0.8,0.9],[0.05,0.1]),(h_3,[0.6,0.7],[0.2,0.25]),(h_4,[0.65,0.78],[0.15,0.21]),(h_5,[0.56,0.68],[0.2,0.3])\}$$

$$F(e_2)=\{(h_1,[0.7,0.8],[0.15,0.21]),(h_2,[0.6,0.7],[0.15,0.21]),(h_3,[0.5,0.7],[0.2,0.3]),(h_4,[0.7,0.75],[0.15,0.25]),(h_5,[0.6,0.7],[0.25,0.31])\}$$

$$F(e_3)=\{(h_1,[0.75,0.85],[0.1,0.15]),(h_2,[0.5,0.6],[0.2,0.35]),(h_3,[0.6,0.8],[0.1,0.18]),(h_4,[0.68,0.75],[0.1,0.2]),(h_5,[0.7,0.8],[0.1,0.21])\}$$

$$F(e_4)=\{(h_1,[0.8,0.9],[0.01,0.1]),(h_2,[0.65,0.75],[0.2,0.25]),(h_3,[0.66,0.77],[0.2,0.22]),(h_4,[0.69,0.78],[0.1,0.2])(h_5,[0.72,0.82],[0.1,0.15])\}$$

$$F(e_5)=\{(h_1,[0.77,0.88],[0.05,0.1]),(h_2,[0.6,0.7],[0.2,0.28]),(h_3,[0.6,0.73],[0.2,0.25]),(h_4,[0.63,0.76],[0.15,0.2]),(h_5,[0.7,0.83],[0.1,0.17])\}$$

The interval valued intuitionistic fuzzy soft set (F,A) is a parameterized family $\{F(e_i, i=1,2,3,4,5)\}$ of interval valued intuitionistic fuzzy sets on U and

Obviously, we can see that the precise evaluation for each object on each parameter is unknown while the lower and upper limits of such an evaluation are given. For example, we can't present the precise membership degree and non membership degree of how expensive house h_1 is, however, house h_1 is at least expensive on the membership degree of 0.6 and it is at most expansive on the membership degree of 0.8; house h_1 is not at least expensive on the non-membership degree of 0.1 and it is not at most expansive on the non-membership degree of 0.2.

Definition 2.6: [3] Let (F_A, E) be a soft set over U. Then a subset of $U \times E$ is uniquely defined by $R_A = \{(u,e) : e \in A, u \in F_A(e)\}$, which is called a relation form of (F_A, E). Now the characteristic function of R_A is written by,

$$\chi_{R_A} : U \times E \to \{0,1\}, \chi_{R_A} = \begin{cases} 1, (u,e) \in R_A \\ 0, (u,e) \notin R_A \end{cases}$$

Let $U = \{u_1, u_2, ..., u_m\}$, $E = \{e_1, e_2, ..., e_n\}$, then R_A can be presented by a table as in the following form

	e_1	e_2	…………..	e_n
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	…………….	$\chi_{R_A}(u_1,e_n)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	……………	$\chi_{R_A}(u_2,e_n)$
……….	……………	……………	……………	…………..
u_m	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	…………….	$\chi_{R_A}(u_m,e_n)$

If $a_{ij} = \chi_{R_A}(u_i,e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

Which is called a soft matrix of order $m \times n$ corresponding to the soft set (F_A, E) over U. A soft set (F_A, E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. Therefore we shall identify any soft set with its soft matrix and use these two concepts as interchangeable.

Example 2.6: Assume that $U = \{u_1, u_2, ..., u_5\}$ be a universal set and $E = \{e_1, e_2, ..., e_5\}$ be a set of all parameters. If $A = \{e_2, e_3, e_4\}$ and $F_A(e_2) = \{u_2, u_4\}$, $F_A(e_3) = \phi$, $F_A(e_4) = U$, then we write a soft set $(F_A, E) = \{(e_2, \{u_2, u_4\}), (e_4, U)\}$ and then the relation form of (F_A, E) is written by,

$R_A = \{(u_2,e_2),(u_4,e_2),(u_1,e_4),(u_2,e_4),(u_3,e_4),(u_4,e_4),(u_5,e_4)\}$.

Hence the soft matrix (a_{ij}) is written by,

$$(a_{ij}) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3. Some new concepts of interval valued intuitionistic fuzzy soft matrices:

3.1 Interval valued intuitionistic fuzzy soft matrix:

Let $\left(F_A^*, E\right)$ be an interval valued intuitionistic fuzzy soft set over U. Then a subset of $U \times E$ is uniquely defined by

$R_A = \left\{(u,e) : e \in A, u \in F_A^*(e)\right\}$, which is called a relation set of $\left(F_A^*, E\right)$. The relation set R_A is characterized by the membership function $\mu_A : U \times E \to Int([0,1])$ and the non-membership function $\gamma_A : U \times E \to Int([0,1])$ (where $Int([0,1])$ is the set of all closed intervals of $[0,1]$) are such that $\forall x \in X, 0 \le \sup \mu_A(x) + \sup \gamma_A(x) \le 1$ is satisfied.

Let $U = \{u_1, u_2, ..., u_m\}$ be the set of universe and $E = \{e_1, e_2, ..., e_n\}$ be the set of parameters, then the relation set R_A can be represented in the following form

	e_1	e_2	…	e_n
u_1	$\left(\left[\mu_{A_L 11}, \mu_{A_U 11}\right]\left[\gamma_{A_L 11}, \gamma_{A_U 11}\right]\right)$	$\left(\left[\mu_{A_L 12}, \mu_{A_U 12}\right]\left[\gamma_{A_L 12}, \gamma_{A_U 12}\right]\right)$	….	$\left(\left[\mu_{A_L 1n}, \mu_{A_U 1n}\right]\left[\gamma_{A_L 1n}, \gamma_{A_U 1n}\right]\right)$
u_2	$\left(\left[\mu_{A_L 21}, \mu_{A_U 21}\right]\left[\gamma_{A_L 21}, \gamma_{A_U 21}\right]\right)$	$\left(\left[\mu_{A_L 22}, \mu_{A_U 22}\right]\left[\gamma_{A_L 22}, \gamma_{A_U 22}\right]\right)$	…	$\left(\left[\mu_{A_L 2n}, \mu_{A_U 2n}\right]\left[\gamma_{A_L 2n}, \gamma_{A_U 2n}\right]\right)$
……	……………	……………	…	…………
u_m	$\left(\left[\mu_{A_L m1}, \mu_{A_U m1}\right]\left[\gamma_{A_L m1}, \gamma_{A_U m1}\right]\right)$	$\left(\left[\mu_{A_L m2}, \mu_{A_U m2}\right]\left[\gamma_{A_L m2}, \gamma_{A_U m2}\right]\right.$	….	$\left(\left[\mu_{A_L mn}, \mu_{A_U mn}\right]\left[\gamma_{A_L mn}, \gamma_{A_U mn}\right]\right)$

Where $\left(\left[\mu_{A_L mn}, \mu_{A_U mn}\right]\left[\gamma_{A_L mn}, \gamma_{A_U mn}\right]\right) = \left(\mu_A(u_m, e_n), \gamma_A(u_m, e_n)\right)$

If $a_{ij} = \left(\left[\mu_{A_L ij}, \mu_{A_U ij}\right]\left[\gamma_{A_L ij}, \gamma_{A_U ij}\right]\right)$, then we can define a matrix

$$\left(a_{ij}^*\right) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

, which is called an interval valued intuitionistic fuzzy soft matrix of order $m \times n$ corresponding to the interval valued intuitionistic fuzzy soft set $\left(F_A^*, E\right)$ over U.

Example 3.1

Let U be the set of four cars given by , $U = \{c_1, c_2, c_3, c_4\}$. Let E be the set of parameters (each parameter is a interval valued intuitionistic fuzzy word) , given by,

E ={ comfortable, color, dimension, price,excellent engine}= $\{e_1, e_2, e_3, e_4, e_5\}$

Let $A \subset E$, given by $A = \{e_1, e_2, e_3, e_4\}$

Suppose that F_A^* is a mapping , defined as attractiveness of the car (.) and given by,

$$F_A^*(e_1) = \left\{c_1 / [0.2, 0.5][0.3, 0.4], c_2 / [0.1, 0.7][0.2, 0.3], c_3 / [0.3, 0.5][0.1, 0.4], c_4 / [0.2, 0.5][0.1, 0.5]\right\}$$

$$F_A^*(e_2) = \left\{c_1 / [0.3, 0.5][0.2, 0.4], c_2 / [0.2, 0.6][0.2, 0.4], c_3 / [0.3, 0.4][0.4, 0.6], c_4 / [0.2, 0.6][0.3, 0.4]\right\}$$

$$F_A^*(e_3) = \left\{c_1 / [0.1, 0.6][0.2, 0.4], c_2 / [0.2, 0.7][0.2, 0.3], c_3 / [0.4, 0.5][0.3, 0.4], c_4 / [0.2, 0.5][0.3, 0.5]\right\}$$

$$F_A^*(e_4) = \left\{c_1 / [0.2, 0.6][0.3, 0.4], c_2 / [0.1, 0.6][0.2, 0.3], c_3 / [0.2, 0.5][0.3, 0.4], c_4 / [0.3, 0.5][0.2, 0.5]\right\}$$

The relation set of $\left(F_A^*, E\right)$ is written as

$$R_A = \left\{ \begin{array}{l} \left(\left\{c_1/[0.2,0.5][0.3,0.4], c_2/[0.1,0.7][0.2,0.3], c_3/[0.3,0.5][0.1,0.4], c_4/[0.2,0.5][0.1,0.5]\right\}, e_1\right), \\ \left(\left\{c_1/[0.3,0.5][0.2,0.4], c_2/[0.2,0.6][0.2,0.4], c_3/[0.3,0.4][0.4,0.6], c_4/[0.2,0.6][0.3,0.4]\right\}, e_2\right), \\ \left(\left\{c_1/[0.1,0.6][0.2,0.4], c_2/[0.2,0.7][0.2,0.3], c_3/[0.4,0.5][0.3,0.4], c_4/[0.2,0.5][0.3,0.5]\right\}, e_3\right), \\ \left(\left\{c_1/[0.2,0.6][0.3,0.4], c_2/[0.1,0.6][0.2,0.3], c_3/[0.2,0.5][0.3,0.4], c_4/[0.3,0.5][0.2,0.5]\right\}, e_4\right) \end{array} \right\}$$

Thus the interval valued intuitionistic fuzzy soft matrix $\left(a_{ij}^*\right)$ is given by,

$$\left(a_{ij}^*\right) = \begin{pmatrix} [0.2.0.5][0.3,0.4] & [0.3,0.5][0.2,0.4] & [0.1,0.6][0.2,0.4] & [0.2,0.6][0.3,0.4] \\ [0.1,0.7][0.2,0.3] & [0.2,0.6][0.2,0.4] & [0.2,0.7][0.2,0.3] & [0.1,0.6][0.2,0.3] \\ [0.3,0.5][0.1,0.4] & [0.3,0.4][0.4,0.6] & [0.4,0.5][0.3,0.4] & [0.2,0.5][0.3,0.4] \\ [0.2,0.5][0.1,0.5] & [0.2,0.6][0.3,0.4] & [0.2,0.5][0.3,0.5] & [0.3,0.5][0.2,0.5] \end{pmatrix}$$

3.2 Row- interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix corresponds to a interval valued intuitionistic fuzzy soft set of order $1 \times n$ i.e. with a single row is called a row-interval valued intuitionistic fuzzy soft matrix.

Reference to Example 3.1 , suppose the universe set U contains only one car c_1 and the parameter set $E = \{e_1, e_2, e_3, e_4, e_5\}$. Let $A = \{e_2, e_3, e_4\} \subset E$ and

$F_A^*(e_2) = \{c_1/[0.1,0.6][0.3,0.4]\}$, $F_A^*(e_3) = \{c_1/[0.2,0.5][0.3,0.5]\}$, $F_A^*(e_4) = \{c_1/[0.3,0.5][0.1,0.4]\}$

Then ,

$$R_A = \left\{\left(\left\{c_1/[0.1,0.6][0.3,0.4]\right\}, e_2\right), \left(\left\{c_1/[0.2,0.5][0.3,0.5]\right\}, e_3\right), \left(\left\{c_1/[0.3,0.5][0.1,0.4]\right\}, e_4\right)\right\}$$

Thus,

$$\left(a_{ij}^*\right) = \left([0,0][1,1] \quad [0.1,0.6][0.3,0.4] \quad [0.2,0.5][0.3,0.5] \quad [0.3,0.5][0.1,0.4]\right)$$

3.3 Column- interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix corresponds to a interval valued intuitionistic fuzzy soft set of order $m \times 1$ i.e. with a single column is called a row-interval valued intuitionistic fuzzy soft matrix.

Reference to Example 3.1 , suppose the universe set $U = \{c_1, c_2, c_3, c_4\}$ and the parameter set E contains only one parameter given by $E = \{e_1\}$. Then,

$F_A^*(e_1) = \left\{(e_1, c_1/[0.3,0.6][0.2,0.4]), (e_1, c_2/[0.1,0.7][0.2,0.3]), (e_1, c_3/[0.3,0.6][0.2,0.4]), (e_1, c_4/[0.1,0.5][0.2,0.4])\right\}$

Therefore,

$$R_A = \left\{\left(\left\{c_1/[0.3,0.6][0.2,0.4], c_2/[0.1,0.7][0.2,0.3], c_3/[0.3,0.6][0.2,0.4], c_4/[0.1,0.5][0.2,0.4]\right\}, e_1\right)\right\}$$

Thus, $\left(a_{ij}^*\right) = \begin{pmatrix} [0.3,0.6][0.2,0.4] \\ [0.1,0.7][0.2,0.3] \\ [0.3,0.6][0.2,0.4] \\ [0.1,0.5][0.2,0.4] \end{pmatrix}$

3.4 Square interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix corresponds to a interval valued intuitionistic fuzzy soft set of order $m \times n$ is called a square interval valued intuitionistic fuzzy soft matrix if $m = n$ i.e number of objects= number of parameters otherwise it is called a rectangular interval valued intuitionistic fuzzy soft matrix.

For illustration, see example 3.1

3.5 Complement of an interval valued intuitionistic fuzzy soft matrix:

Let $\left(a_{ij}^{*}\right)$ be an $m \times n$ interval valued intuitionistic fuzzy soft matrix, where $a_{ij}^{*} = \left(\left[\mu_{L}ij, \mu_{U}ij\right]\left[\gamma_{L}ij, \gamma_{U}ij\right]\right)$ $\forall\, i, j$. Then the complement of $\left(a_{ij}^{*}\right)$ is denoted by $\left(\overline{a_{ij}^{*}}\right)$ and is defined by,

$\left(\overline{a_{ij}^{*}}\right) = \left(b_{ij}^{*}\right)$, where $\left(b_{ij}^{*}\right)$ is also an interval valued intuitionistic fuzzy soft matrix of order $m \times n$ and $b_{ij}^{*} = \left(\left[1-\mu_{Uij}, 1-\mu_{L}ij\right]\left[1-\gamma_{Uij}, 1-\gamma_{L}ij\right]\right)$ $\forall\, i, j.$

For illustration, by using example 3.1, we have

$$\left(b_{ij}^{*}\right) = \begin{pmatrix} [0.5.0.8][0.6,0.7] & [0.5,0.7][0.6,0.8] & [0.4,0.9][0.6,0.8] & [0.4,0.8][0.6,0.7] \\ [0.3,0.9][0.7,0.8] & [0.4,0.8][0.6,0.8] & [0.3,0.8][0.7,0.8] & [0.4,0.9][0.7,0.8] \\ [0.5,0.7][0.6,0.9] & [0.6,0.7][0.4,0.6] & [0.5,0.6][0.6,0.7] & [0.5,0.8][0.6,0.7] \\ [0.5,0.8][0.5,0.9] & [0.4,0.8][0.6,0.7] & [0.5,0.8][0.5,0.7] & [0.5,0.7][0.5,0.8] \end{pmatrix}$$

3.6 Null or Zero interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a null or zero interval valued intuitionistic fuzzy soft matrix if all of its elements are $([0,0],[1,1])$ and it is denoted by the symbol ϕ^{*}.

3.7 Complete or Absolute interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a complete or absolute interval valued intuitionistic fuzzy soft matrix if all of its elements are $([1,1],[0,0])$ and it is denoted by the symbol C_{A}^{*}.

3.8 Diagonal interval valued intuitionistic fuzzy soft matrix:

An interval valued intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a diagonal interval valued intuitionistic fuzzy soft matrix if all of its non diagonal elements are $([0,0],[1,1])$.

3.9 Transpose of an interval valued intuitionistic fuzzy soft matrix:

The transpose of a square interval valued intuitionistic fuzzy soft matrix of order $m \times n$ is said to be a square interval valued intuitionistic fuzzy soft matrix of order $n \times m$ and it is obtained by interchanging its rows and columns and it is denoted by $\left(a_{ij}^{*}\right)^{T}$.

3.10 Trace of an interval valued intuitionistic fuzzy soft matrix:

Trace of an interval valued intuitionistic fuzzy soft matrix $\left(a_{ij}^{*}\right)$ is denoted by $tr\left(a_{ij}^{*}\right)$ and to define it first we determine $\left(\left[\max \mu_{L}ij, \max \mu_{U}ij\right]\left[\min \gamma_{L}ij, \min \gamma_{U}ij\right]\right)$ where $i = j$. Then,

$tr\left(a_{ij}^{*}\right) = \max \mu_{L}ij \times \min \gamma_{L}ij + \max \mu_{U}ij \times \min \gamma_{U}ij$ and its value belongs to the interval $[0,1]$.

3.11 Symmetric interval valued intuitionistic fuzzy soft matrix:

A square interval valued intuitionistic fuzzy soft matrix $\left(a_{ij}^{*}\right)$ of order $n \times n$ is said to be symmetric if $\left(a_{ij}^{*}\right)^{T} = \left(a_{ij}^{*}\right)$.

Note: In our discussion there is no scope to define skew symmetric interval valued intuitionistic fuzzy soft matrix.

3.12 Scalar multiple of an interval valued intuitionistic fuzzy soft matrix:

For any scalar $k \in [0,1]$, the scalar multiple of an interval valued intuitionistic fuzzy soft matrix $\left(a_{ij}^{*}\right)$ is denoted by $k\left(a_{ij}^{*}\right)$ and it is defined as $k\left(a_{ij}^{*}\right) = \left([k\mu_L ij, k\mu_U ij][k\gamma_L ij, k\gamma_U ij]\right)\ \forall\, i, j.$

3.13 Choice matrix with an interval valued intuitionistic fuzzy soft set:

A matrix whose rows and columns both indicate parameters is called a choice matrix. If η is a choice matrix, then its elements η_{ij} is defined as follows:

$\eta_{ij} = ([1,1],[0,0])$, when i-th and j-th parameters both choice parameters of the decision makers.

$\quad = ([0,0],[1,1])$, when atleast one of the i-th or j-th parameters not under choice.

Choice matrices differ according to the number of decision makers.

Example 3.13

Let U be a set of four dresses and let E be a set of parameters given by,

$U = \{d_1, d_2, d_3, d_4\}$ *and* $E = \{beautiful, cheap, comfortable, gorgeous\} = \{e_1, e_2, e_3, e_4\}$

Now let the interval valued intuitionistic fuzzy soft set $\left(F_A^*, E\right)$ describing the attractiveness of the dresses is given by,

$$\left(F_A^*, E\right) = \begin{Bmatrix} \textit{beautiful dresses} = \{d_1/[0.3,0.6][0.2,0.4], d_2/[0.1,0.7][0.2,0.3], d_3/[0.2,0.5][0.3,0.4], d_4/[0.2,0.7][0.1,0.3]\}, \\ \textit{cheap dresses} = \{d_1/[0.4,0.5][0.2,0.4], d_2/[0.3,0.5][0.2,0.4], d_3/[0.6,0.7][0.2,0.3], d_4/[0.5,0.6][0.2,0.3]\}, \\ \textit{comfortable dresses} = \{d_1/[0.3,0.4][0.4,0.5], d_2/[0.2,0.3][0.5,0.6], d_3/[0.1,0.2][0.7,0.8], d_4/[0.5,0.6][0.1,0.2]\}, \\ \textit{gorgeous dresses} = \{d_1/[0.3,0.4][0.4,0.5], d_2/[0.5,0.6][0.1,0.2], d_3/[0.5,0.7][0.2,0.3], d_4/[0.6,0.8][0.1,0.2]\} \end{Bmatrix}$$

Suppose Mr.Trump wants to buy a dress on the basis of his choice parameters which form a subset of E

and it is $X = \{e_1, e_2, e_4\}$.

Therefore, the choice matrix of Mr.Trump is,

$$\left(\eta_{ij}\right)_X = \begin{array}{cc} & e_X \\ e_X & \begin{pmatrix} [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

Again, suppose Mr. Trump and Miss. Trump together wants to buy a dress according to their choice parameters. Let the choice parameter set of Miss. Trump be,

$Y = \{e_1, e_2, e_3, e_4\}$

Then the combined choice matrix of Mr. Trump and Miss. Trump is

$$\left(\eta_{ij}\right)_{(X,Y)} = \begin{array}{cc} & e_Y \\ e_X & \begin{pmatrix} [1,1][0,0] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}$$

The above combined matrix can also be represented in its transpose form.

Suppose we are thinking of another combined matrix associated with three decision makers. For this let Jr.Trump is willing to buy a dress together with Mr. Trump and Miss. Trump on the basis of his choice parameter set $Z = \{e_2, e_3, e_4\}$.

Then the combined choice matrix of Mr. Trump, Miss. Trump and Jr.Trump will be of three different types which are as follows,

i) $$\left(\eta_{ij}\right)_{(Z,X\wedge Y)} = \begin{array}{cc} & e_{X\wedge Y} \\ e_Z & \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

ii) $$\left(\eta_{ij}\right)_{(X,Y\wedge Z)} = \begin{array}{cc} & e_{Y\wedge Z} \\ e_X & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}$$

iii) $$\left(\eta_{ij}\right)_{(Y,Z\wedge X)} = \begin{array}{cc} & e_{Z\wedge X} \\ e_Y & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

3.14 Product of an interval valued intuitionistic fuzzy soft matrix with a choice matrix:

Let U be the set of universe and E be the set of parameters.Suppose that A be any interval valued intuitionistic fuzzy soft matrix and C be any choice matrix of a decision maker concerned with the same U and E. Then the product of A and C is denoted by $A\square\ C$ or simply AC.

If $A=\left(a_{ij}^{*}\right)_{m\times n}$ and $C=\left(c_{jk}^{*}\right)_{n\times p}$, then $AC=\left(d_{ik}^{*}\right)$

where

$$d_{ik}^{*}=\left(\left[\max_{j=1}^{n}\min\left\{\mu_{La_{ij}^{*}},\mu_{Lc_{jk}^{*}}\right\},\max_{j=1}^{n}\min\left\{\gamma_{Ua_{ij}^{*}},\gamma_{Uc_{jk}^{*}}\right\}\right],\left[\min_{j=1}^{n}\max\left\{\mu_{La_{ij}^{*}},\mu_{Lc_{jk}^{*}}\right\},\min_{j=1}^{n}\max\left\{\gamma_{Ua_{ij}^{*}},\gamma_{Uc_{jk}^{*}}\right\}\right]\right)$$

But CA cannot be defined here.

Example 3.14

Reference to example 3.13, suppose that the set of choice parameters of Mr. Trump be $A=\{e_1,e_3\}$.

Now let according to the choice parameters of Mr. Trump, we have the following interval valued intuitionistic fuzzy soft matrix corresponding to the interval valued intuitionistic fuzzy soft set $\left(F_A^{*},E\right)$.

$$\left(a_{ij}^{*}\right)=\begin{pmatrix} [0.3,0.6][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5] \\ [0.1,0.7][0.2,0.3] & [0.3,0.5][0.2,0.4] & [0.2,0.3][0.5,0.6] & [0.5,0.6][0.1,0.2] \\ [0.2,0.5][0.3,0.4] & [0.6,0.7][0.2,0.3] & [0.1,0.2][0.7,0.8] & [0.5,0.7][0.2,0.3] \\ [0.2,0.7][0.1,0.3] & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

Again the choice matrix of Mr. Trump is,

$$\left(\eta_{ij}\right)_A = \begin{array}{cc} & e_A \\ e_A & \begin{pmatrix} [1,1][0,0] & [0,0][1,1] & [1,1][0,0] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [0,0][1,1] & [1,1][0,0] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \end{pmatrix} \end{array}$$

Then their product is given by

$$\begin{pmatrix} [0.3,0.6][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5] \\ [0.1,0.7][0.2,0.3] & [0.3,0.5][0.2,0.4] & [0.2,0.3][0.5,0.6] & [0.5,0.6][0.1,0.2] \\ [0.2,0.5][0.3,0.4] & [0.6,0.7][0.2,0.3] & [0.1,0.2][0.7,0.8] & [0.5,0.7][0.2,0.3] \\ [0.2,0.7][0.1,0.3] & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \square \begin{pmatrix} [1,1][0,0] & [0,0][1,1] & [1,1][0,0] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [0,0][1,1] & [1,1][0,0] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \end{pmatrix}$$

$$= \begin{pmatrix} [0.3,0.6][0.2,0.4] & [0,0][1,1] & [0.3,0.6][0.2,0.4] & [0,0][1,1] \\ [0.2,0.7][0.2,0.3] & [0,0][1,1] & [0.2,0.7][0.2,0.3] & [0,0][1,1] \\ [0.2,0.5][0.3,0.4] & [0,0][1,1] & [0.2,0.5][0.3,0.4] & [0,0][1,1] \\ [0.5,0.7][0.1,0.2] & [0,0][1,1] & [0.5,0.7][0.1,0.2] & [0,0][1,1] \end{pmatrix}$$

4. A new technique to solve interval valued intuitionistic fuzzy soft set based decision making problems:

Here we introduce a new technique which is basically based on choice matrices.Choice matrices represent the choice parameters of the decision makers and it also help us to solve the interval valued intuitionistic fuzzy soft matrix based decision making problems with least computational complexity.Now at first we consider a generalized interval valued intuitionistic fuzzy soft set based decision making problem.

4.1 Generalized interval valued intuitionistic fuzzy soft set based decision making problem:

Suppose $U=\{O_1,O_2,O_3,......O_m\}$ be the set of m objects and $E=\{e_1,e_2,e_3,......e_n\}$ be the set of n parameters.Now let N number of decision makers $D_1,D_2,.......,D_N$ want to select an object from U according to their set of choice parameters $P_{D_1},P_{D_2},........,P_{D_N}\subseteq E$ .Now the problem is to find out the optimal object from U which satisfies all of these choice parameters of the decision makers as much as possible.

4.2 The stepwise solving procedure:

We consider the following stepwise procedure to solve such type of problems.

Algorithm:

Step-1: First construct the combined choice matrix with respect to the choice parameters of the decision makers.

Step-2: Compute the product interval valued intuitionistic fuzzy soft matrices by multiplying each given interval valued intuitionistic fuzzy soft matrix with the combined choice matrix as per the rule of multiplication of interval valued intuitionistic fuzzy soft matrices.

Step-3: Compute the sum of these product interval valued intuitionistic fuzzy soft matrices to have the resultant interval valued intuitionistic fuzzy soft matrix (R_f).

Step-4: Compute the weight of each object (O_i) by adding the product of the maximum membership values and the minimum non-membership values of the entries of its concerned row $(i-th\ row)$ of (R_f) and it is denoted by $W(O_i)$.

Step-5: The object having the highest weight becomes the optimal choice object. If more than one object have the highest weight then go to next step.

Step-6: Now we consider the sum of the product of the minimum membership values and the maximum non-membership values of the entries of its concerned row $(i-th\ row)$ of (R_f) and it is denoted by Θ . The object with the minimum Θ value will be the optimal choice object. If there is a tie then any one of them may be chosen as the optimal choice object.

Example 4.1

Reference to the example 3.13, suppose Mr. Trump, Miss. Trump and Jr.Trump together want to buy a dress among these four dresses for their common friend Mr. Obama according their choice parameters $X=\{e_1,e_2,e_4\}$, $Y=\{e_1,e_2,e_3,e_4\}$, $Z=\{e_2,e_3,e_4\}$ respectively.

Now let according to the choice parameters of Mr. Trump, Miss. Trump and Jr. Trump , we have the interval valued intuitionistic fuzzy soft sets $\left(F_A^*, E\right)$ which describe the attractiveness of the dresses .

The combined choice matrices of Mr. Trump, Miss. Trump and Jr.Trump in different forms are

$$\begin{array}{cc} & e_{Y\wedge Z} \\ e_X & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}, \begin{array}{cc} & e_{Z\wedge X} \\ e_Y & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array} \text{ and}$$

$$\begin{array}{cc} & e_{X\wedge Y} \\ e_Z & \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

Corresponding product interval valued intuitionistic fuzzy soft matrices are

$$\text{i)} \begin{pmatrix} [0.3,0.6][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5] \\ [0.1,0.7][0.2,0.3] & [0.3,0.5][0.2,0.4] & [0.2,0.3][0.5,0.6] & [0.5,0.6][0.1,0.2] \\ [0.2,0.5][0.3,0.4] & [0.6,0.7][0.2,0.3] & [0.1,0.2][0.7,0.8] & [0.5,0.7][0.2,0.3] \\ [0.2,0.7][0.1,0.3] & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \square \begin{array}{cc} & e_{Y\wedge Z} \\ e_X & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [1,1][0,0] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} [0,0][1,1] & [0.4,0.6][0.2,0.4] & [0.4,0.6][0.2,0.4] & [0.4,0.6][0.2,0.4] \\ [0,0][1,1] & [0.5,0.7][0.1,0.2] & [0.5,0.7][0.1,0.2] & [0.5,0.7][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$\text{ii)} \begin{pmatrix} [0.3,0.6][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5] \\ [0.1,0.7][0.2,0.3] & [0.3,0.5][0.2,0.4] & [0.2,0.3][0.5,0.6] & [0.5,0.6][0.1,0.2] \\ [0.2,0.5][0.3,0.4] & [0.6,0.7][0.2,0.3] & [0.1,0.2][0.7,0.8] & [0.5,0.7][0.2,0.3] \\ [0.2,0.7][0.1,0.3] & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \square \begin{array}{cc} & e_{Z\wedge X} \\ e_Y & \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} [0,0][1,1] & [0.4,0.6][0.2,0.4] & [0,0][1,1] & [0.4,0.6][0.2,0.4] \\ [0,0][1,1] & [0.5,0.7][0.1,0.2] & [0,0][1,1] & [0.5,0.7][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$\text{iii)} \begin{pmatrix} [0.3,0.6][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.3,0.4][0.4,0.5] & [0.3,0.4][0.4,0.5] \\ [0.1,0.7][0.2,0.3] & [0.3,0.5][0.2,0.4] & [0.2,0.3][0.5,0.6] & [0.5,0.6][0.1,0.2] \\ [0.2,0.5][0.3,0.4] & [0.6,0.7][0.2,0.3] & [0.1,0.2][0.7,0.8] & [0.5,0.7][0.2,0.3] \\ [0.2,0.7][0.1,0.3] & [0.5,0.6][0.2,0.3] & [0.5,0.6][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \square \begin{array}{cc} & e_{X\wedge Y} \\ e_Z & \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$= \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

The sum of these products

$$\begin{pmatrix} [0,0][1,1] & [0.4,0.6][0.2,0.4] & [0.4,0.6][0.2,0.4] & [0.4,0.6][0.2,0.4] \\ [0,0][1,1] & [0.5,0.7][0.1,0.2] & [0.5,0.7][0.1,0.2] & [0.5,0.7][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \oplus \begin{pmatrix} [0,0][1,1] & [0.4,0.6][0.2,0.4] & [0,0][1,1] & [0.4,0.6][0.2,0.4] \\ [0,0][1,1] & [0.5,0.7][0.1,0.2] & [0,0][1,1] & [0.5,0.7][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$\oplus \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$= \begin{pmatrix} [0.4, 0.5][0.2, 0.4] & [0.4, 0.6][0.2, 0.4] & [0.4, 0.6][0.2, 0.4] & [0.4, 0.6][0.2, 0.4] \\ [0.5, 0.6][0.1, 0.2] & [0.5, 0.7][0.1, 0.2] & [0.5, 0.7][0.1, 0.2] & [0.5, 0.7][0.1, 0.2] \\ [0.6, 0.7][0.2, 0.3] & [0.6, 0.7][0.2, 0.3] & [0.6, 0.7][0.2, 0.3] & [0.6, 0.7][0.2, 0.3] \\ [0.6, 0.8][0.1, 0.2] & [0.6, 0.8][0.1, 0.2] & [0.6, 0.8][0.1, 0.2] & [0.6, 0.8][0.1, 0.2] \end{pmatrix} = R_f$$

Now the weights of the dresses are,

$W(d_1) = 0.1 + 0.12 + 0.12 + 0.12 = 0.46$

$W(d_2) = 0.06 + 0.07 + 0.07 + 0.07 = 0.27$

$W(d_3) = 0.14 + 0.14 + 0.14 + 0.14 = 0.56$

$W(d_4) = 0.08 + 0.08 + 0.08 + 0.08 = 0.32$

From above it is clear that the dress associated with the third row of the resultant matrix R_f has the highest weight i.e, $W(d_3) = 0.56$. Thus d_3 is the optimal choice dress.

5. Predicting election results

There are many factors that influence the voters to vote at the ballot box.One of the key factors is the personal attributes of the candidate ,who is the person running for a political purpose , are of the most important characteristics that a person takes into account. Another is the political party to which the candidate belongs. There are also several important factors that how an individual votes. It is these factors and how they influence election results. For the following example,we consider three groups of people namely X,Y and Z to predict election results on the basis of one of these factors by using the algorithm discussed in section 4.2 .

Example 5.1

Let U be the set of political leaders and it is given as $U = \{P_1, P_2, P_3, P_4\}$ and E be the set of parameters written as $E = \{\text{patriotism, honesty, intelligence, responsibility}\} = \{e_1, e_2, e_3, e_4\}$.

Now suppose that X,Y and Z decided to select a political leader according to their corresponding choice parameters set.

Let the choice parameters set of groups X,Y and Z are respectively,

$A = \{e_1, e_2, e_4\}$, $B = \{e_1, e_3, e_4\}$ and $C = \{e_2, e_3, e_4\}$

Now let the interval valued intuitionistic fuzzy soft matrices associated with the interval valued intuitionistic fuzzy soft sets $(F_A^*, E), (F_B^*, E)$ and (F_C^*, E) describing the personality of the political leaders which predict the election results according to the groups of people X,Y and Z respectively.

$$\left(a_{ij}^*\right) = \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$\left(b_{ik}^*\right) = \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.3,0.4] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.3,0.6][0.3,0.4] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.2,0.8][0.1,0.2] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.1,0.7][0.2,0.3] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$\left(c_{il}^{*}\right)=\begin{pmatrix} [0,0][1,1] & [0.3,0.4][0.5,0.6] & [0.4,0.5][0.3,0.4] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0.2,0.6][0.3,0.4] & [0.3,0.6][0.3,0.4] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0.3,0.5][0.2,0.4] & [0.2,0.8][0.1,0.2] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.5,0.6][0.3,0.4] & [0.1,0.7][0.2,0.3] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

Now the problem is to find the political leader which is most favourite among these four for winning election from all the groups X,Y and Z.

The combined choice matrices of X,Y and Z in different forms are,

$$e_{A}\begin{array}{c} e_{B\wedge C} \\ \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}, \quad e_{B}\begin{array}{c} e_{C\wedge A} \\ \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$e_{C}\begin{array}{c} e_{A\wedge B} \\ \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

Corresponding product interval valued intuitionistic fuzzy soft matrices are,

$$U_{A}\begin{array}{c} e_{A} \\ \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix} \end{array} \square \; e_{A}\begin{array}{c} e_{B\wedge C} \\ \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [0,0][1,1] & [1,1][0,0] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$=\begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0,0][1,1] & [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$U_{B}\begin{array}{c} e_{B} \\ \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.3,0.4] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.3,0.6][0.3,0.4] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.2,0.8][0.1,0.2] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.1,0.7][0.2,0.3] & [0.6,0.8][0.1,0.2] \end{pmatrix} \end{array} \square \; e_{B}\begin{array}{c} e_{C\wedge A} \\ \begin{pmatrix} [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \\ [0,0][1,1] & [1,1][0,0] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$=\begin{pmatrix} [0,0][1,1] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$U_{C}\begin{array}{c} e_{C} \\ \begin{pmatrix} [0,0][1,1] & [0.3,0.4][0.5,0.6] & [0.4,0.5][0.3,0.4] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0.2,0.6][0.3,0.4] & [0.3,0.6][0.3,0.4] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0.3,0.5][0.2,0.4] & [0.2,0.8][0.1,0.2] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.5,0.6][0.3,0.4] & [0.1,0.7][0.2,0.3] & [0.6,0.8][0.1,0.2] \end{pmatrix} \end{array} \square \; e_{C}\begin{array}{c} e_{A\wedge B} \\ \begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0,0][1,1] & [0,0][1,1] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \\ [1,1][0,0] & [0,0][1,1] & [0,0][1,1] & [1,1][0,0] \end{pmatrix} \end{array}$$

$$=\begin{pmatrix} [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

The sum of these product interval valued intuitionistic fuzzy soft matrices is,

$$\begin{pmatrix} [0,0][1,1] & [0,0][1,1] & [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0,0][1,1] & [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix} \oplus \begin{pmatrix} [0,0][1,1] & [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0,0][1,1] & [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0,0][1,1] & [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0,0][1,1] & [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix} \oplus \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0,0][1,1] & [0,0][1,1] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0,0][1,1] & [0,0][1,1] & [0.5,0.6][0.1,0.2] \\ [0.6,0.7][0.2,0.3] & [0,0][1,1] & [0,0][1,1] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0,0][1,1] & [0,0][1,1] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

$$= \begin{pmatrix} [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] & [0.4,0.5][0.2,0.4] \\ [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] & [0.5,0.6][0.1,0.2] \\ 0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] & [0.6,0.7][0.2,0.3] \\ [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] & [0.6,0.8][0.1,0.2] \end{pmatrix}$$

Now the weights of the political leaders are respectively,

$W(P_1)$=0.1+0.1+0.1+0.1=0.4

$W(P_2)$=0.06+0.06+0.06+0.06=0.24

$W(P_3)$=0.14+0.14+0.14+0.14=0.56

$W(P_4)$=0.08+0.08+0.08+0.08=0.32

From above it is clear that P_3 has the highest weight. So P_3 wins the election.

6. Conclusion

In this paper first we have proposed the concept of interval valued intuitionistic fuzzy soft matrix and defined various types of matrices in interval valued intuitionistic fuzzy soft set theory with examples.

At the end a new relevant solution procedure has been developed to solve interval valued intuitionistic fuzzy soft set based real life decision making problems which may contain more than one decision maker and to realize it in more effective way we also apply it to predict election results.

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